

## **Minerals**

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# **Pumping Non-Newtonian Slurries**

## Introduction

Many slurries in the mineral processing industries can be described as being non-Newtonian.

If a fluid is in motion, shear stresses are developed when particles of the fluid move relative to each other. For successive layers of flow over a flat plate, the velocity of the fluid ( $\mu$ ) will vary from layer to layer as distance from the plate (y) increases, see Figure 1. Newton's law of viscosity relates shear stress ( $\tau$ ) to the velocity gradient, also known as the rate of shear strain, or simply the shear rate (du/dy), using a constant of proportionality known as the dynamic viscosity ( $\mu$ ) of the fluid, as follows:

$$\tau = \mu \frac{du}{dy}$$

Fluids which do not obey Newton's law of viscosity are known as non-Newtonian. The relationship of shear stress to shear rate for non-Newtonian fluids can be usually represented by one of the models shown in Figure 2.

Bingham Plastic behaviour is characterised by its initial shear or yield stress ( $\tau_0$ ). This means that the fluid will not flow unless a stress greater than the yield is applied. The slope of the linear flow curve is called the plastic viscosity ( $\eta$ ), also known



Figure 1. Variation in fluid velocity with distance from a flat plate

as coefficient of rigidity. Bingham Plastic behaviour is described by the equation:

$$\tau = \tau_0 + \eta \, \frac{\mathrm{d}u}{\mathrm{d}y}$$

Both centrifugal and positive displacement slurry pumps are often used to handle slurries which behave as non-Newtonian fluids. Many of these slurries can be described by the Bingham Plastic model. Even slurries with different rheological characteristics can sometimes be approximated by the Bingham Plastic model over a limited range of shear rates. Table 1 gives specific examples of some Bingham Plastic slurries.

This bulletin outlines some basic rheological theory, describes (shear stress) / (shear rate) measurement, and presents some empirical relationships relevant to applying centrifugal slurry pumps to a Bingham Plastic slurry. It should be noted that if slurry behaves as a Newtonian fluid, it may be treated as if equivalent to water with appropriate head and efficiency corrections applied.

## **Measurement of viscosity**

The viscosity of a fluid (or more correctly shear stress versus shear rate) cannot be measured directly, but can be calculated from measured



Figure 2. Time-independent fluid flow models



Warman® AHF, MF and LF horizontal pumps best suited for pumping non-Newtonian slurries



Figure 3. Slump plate test

#### **Table 1 - Examples of Bingham Plastic Slurries**

Slurry	Cw (%)	Particle Size d50 (µm)	<b>τ</b> <sub>0</sub> (Pa)	η (Pa·s)
Fine Coal	49	40	1	0.005
Fine Coal	68	40	8.3	0.04
Fly Ash	64	70	8.84	1.304
Kaolin Clay	32	0.8	20	0.03
Kaolin (+ Sod. Silicate)	53	0.8	6	0.015
Kimberlite Tails	37	15	11.6	0.006
KimberliteTails	69	300	132	0.20
Phosphate Tails	37	10	28.5	0.014
Copper Concentrate	48	35	19	0.018
Copper Concentrate	64	35	0.45	0.031
Zinc Concentrate	75	20	12	0.031
Uranium Tails	58	38	4	0.015
GoldTails	50	50	5	0.087
Minerals Sands Tails	55	160	30	0.25
CoalTails	31	70	2	0.06
Red Mud	39	150	23	0.03
Red Mud	50	30	33.2	0.304
Red Mud	53	3	80	0.08

quantities. One apparatus used to make such measurements is generally known as a viscometer. Many types of viscometers are available. A typical rotary viscometer is illustrated in Figure 4. This viscometer shows a concentric cylinder (bob) within another concentric cylinder (cup). The gap between the two cylinders is filled with slurry and the torque required to achieve constant relative velocity between the two cylinders is measured at a number of speeds. From the torque, geometry (diameter, surface area and gap) and rotational speed, the shear stress (N/m<sup>2</sup> or Pa) and, the shear rate [(m/s)/m or s<sup>-1</sup>] can be plotted, as shown in Figure 5 which contains data obtained from testing of a red mud slurry. From this rheogram, or plot of  $\tau$  versus shear rate, the yield stress,  $\tau_0$ , can be estimated as the y-axis intercept. Rotary viscometers can also be used with vane or "paddle" type sensors to obtain the yield stress without the need for extrapolation.

## **Apparent viscosity**

The difference between apparent viscosity ( $\mu_a$ ), which is viscosity at a given rate of shear assuming Newtonian behaviour, and the plastic viscosity ( $\eta$ ), should be emphasised. This difference is illustrated in Figure 5. A Bingham Plastic slurry will have an infinite number of apparent viscosities, depending on the particular rate of shear considered. Accordingly measurements should never be taken at a single rate of



Figure 4. Typical diagram of a rotary viscometer



Figure 5. Behaviour of a red mud slurry calculated from rotary viscometer measurements

shear, and Newtonian behaviour assumed, as the results may be misleading. Apparent viscosity will only approach the plastic viscosity at high shear rates.

## **Slump Plate Testing**

The slump plate test depicted in Figure 3 provides a useful indication of slurry rheology. The test itself is very simple. The plate has a number of graduated rings marked on it. A small cylinder is placed in the center of the plate concentric to the rings. The cylinder is filled with slurry and then removed. The ring to which the slurry slumps indicates the yield stress,  $\tau_0$ , of the slurry. The greater the slump the lower the yield stress. This slump plate testing is typically used to indicate whether a slurry is likely to present difficulty in pumping and at what concentration.

## **System Characteristics**

To apply a Warman<sup>®</sup> centrifugal slurry pump or GEHO<sup>®</sup> positive displacement pump handling a Bingham Plastic slurry, accurate calculation of the pipe friction loss is required. It should be noted that SI units are assumed throughout this bulletin.

The pipe friction loss will depend on whether the flow is in the laminar or turbulent regime. It is generally preferable to operate in the laminar regime, unless there is possibility of solids settlement (refer to the Weir Minerals Slurry Pumping Handbook, or enquire at your nearest Weir Minerals office, for calculation of limiting settling pipeline velocity).

The critical velocity Vc (m/s), also called transition velocity, is defined as the pipe velocity at which transition occurs from laminar to turbulent flow. Experimentally obtained critical velocities for a wide range of Bingham Plastic slurries are plotted against  $\sqrt{(\tau_0/\rho_m)}$  in Figure 6. The following relationship can be established:

 $Vc = 0.4 + 22.1 \bullet \sqrt{(\tau_0 / \rho_m)}$ 

where :  $\tau_0 =$  yield stress (Pa)  $\rho_m =$  slurry density (kg/m<sup>3</sup>)



Figure 6. Critical pipe velocity for Bingham Plastic Slurries

This relationship allows calculation of the critical velocity with only the yield stress and the slurry density. Using the critical velocity, it can be determined if flow is in the laminar, transitional or turbulent regime.

Operation in the laminar flow regime, at or near the critical or transition velocity, can often minimise the chance of settling while keeping the friction lower than operation in the turbulent flow regime.

#### Pipe friction loss with laminar flow

The laminar pipe friction loss can be determined most easily from the Buckingham equation, neglecting 4th order terms. The pressure drop (P) per meter length (L) of pipe is given by:

 $P/L = (5.33\tau_0/D) + (32\eta \cdot V/D^2)$ 

where: V = pipeline velocity (m/s) D = inside pipe diameter (m)  $\eta$  = plastic viscosity (Pa • s)

or in terms of head loss (H):

 $H = P/(g \bullet \rho_m)$ 

where:

g = gravitational constant (m/s<sup>2</sup>)

Alternatively, the pressure drop for large diameter pipelines may be scaled up directly from small bore tube viscometer results. The wall shear stress (PD/4L) can be determined from the rheogram, with the rate of shear given by 8V/D.

#### Pipe friction loss with turbulent flow

The rheological properties of Bingham Plastic slurries do not greatly influence the pipe friction loss in fully developed turbulent flow. Turbulence effectively destroys the yield structure, causing the slurry to behave as a Newtonian fluid with viscosity roughly equal to the plastic viscosity. At high velocities the pressure drop is similar to that for Newtonian fluids (Figure 7). For many commercial slurries the friction loss, expressed in equivalent units of head, can be taken to be 1.1 times the friction loss for water. This should give a conservative estimate of friction loss.



Figure 7. Pipe friction characteristics for a uranium ore slurry (compared with water.)

## **Centrifugal Pump Performance**

Unlike Weir GEHO<sup>®</sup> positive displacement pumps, the clear water performance of a centrifugal pump is affected when handling non-Newtonian slurry as shown in Figure 8. At low flows (generally less than 20% of the best efficiency point flow) the developed head on slurry may be reduced compared to the water head. Experience suggests that this only occurs for small pumps (less than 150mm inlet size) and also may be related to air entrained in slurry. This can prevent pumps from achieving the full system performance. Field and lab test work has shown that the Warman<sup>®</sup> froth pump shown on the first page, has largely overcome this particular problem with negligible effects on head when handling slurries with yield stresses up to 200Pa without a booster pump.

Pump efficiency on slurry may be reduced compared with the water efficiency over the complete flow range. This is not restricted to small pumps. It is due to the increased apparent viscosity. Performance has been found to correlate well with a modified version of the pump Reynolds Number (Re<sub>n</sub>) as follows:

 $\text{Re}_{p} = \omega \bullet D_{i}^{2} \bullet \rho_{m}/\eta$ 

#### where:

 $\omega$  = angular velocity (rad/s, or s<sup>-1</sup>) D<sub>i</sub> = impeller diameter (m)

When  $\text{Re}_{p}$  is less than 1 x 10<sup>6</sup>, efficiency is generally significantly reduced.

## **Suction Performance**

When pumping Bingham Plastic slurries, centrifugal pump suction performance is likely to be affected as suggested by some studies. Experimental data show that NPSH required by the pump increases as compared to the NPSH required on water. The test results obtained for smaller pumps (up to 150mm inlet size) handling materials as diverse as peat and magnetite slurries can be expressed approximately as follows:

NPSHr<sub>p</sub> = NPSHr<sub>w</sub> • (1+24 • Vc •  $c_1/W_1^2$ )

#### where:

NPSHr<sub>p</sub> = NPSHr on Bingham Plastic slurry NPSHr<sub>w</sub> = NPSHr on water c<sub>1</sub> = average inlet velocity (m/s) w<sub>1</sub> =  $\sqrt{(c_1^2 + u_1^2)}$ , relative inlet velocity (m/s) u<sub>1</sub> = impeller inlet peripheral velocity (m/s)

It is expected that with an increase in pump inlet size, the increase in NPSHr will become progressively smaller.



Figure 8. Performance correction factors for special froth pump in comparison with standard centrifugal slurry pump.

## Conclusion

Pump users often require centrifugal slurry pumps to handle slurries which behave as non-Newtonian fluids. To successfully apply a centrifugal slurry pump, both system and the pump performance characteristics must be adjusted to take into account the slurry rheology.

Weir Minerals is continuing to research the influence of slurry rheology on system and pump performance in order to better understand the effects of slurry viscosity on pump reliability. The goal is always to minimise the customer's Total Ownership Costs (TOCs).

## **Weir Minerals**

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